Available online at www.sciencedirect.com
ScienceDirect
Journal of Applied Mathematics and Mechanics 71 (2007) 747–754
Journal of APPLIED MATHEMATICS AND MECHANICS
www.elsevier.com/locate/jappmathmech

# An estimate of the heat fluxes to the surface of blunt bodies moving at hypersonic velocity in the atmosphere☆

V.A. Andrushchenko, N.G. Syzranova, Yu.D. Shevelev

*Moscow, Russia*

Received 24 January 2006

**Abstract**

The problem of the chemically equilibrium three-dimensional boundary layer on a blunt body which is in motion in the atmosphere is considered. A solution of the system of equations of the boundary layer is found by the method of successive approximations, and simple analytic expressions are written in the first approximation for the surface friction and heat flux coefficients. Formulae are obtained in the final form for estimating the convective heat flux in the neighbourhood of the critical point of spherical blunting.
© 2007 Elsevier Ltd. All rights reserved.

As is well known, from the Navier–Stokes equations one can obtain, within the framework of an asymptotic analysis, all possible continual models for the stationary problems of supersonic flow around blunt bodies over the whole range of Reynolds numbers from low (free-molecular and transition flow regimes) to high (flow regimes with a thin head shock wave, a boundary layer and external inviscid flow in a shock layer).[1,2] In this paper we investigate the second of these regimes, for which, on the descending part of the trajectory of the body at altitudes of less than 60–80 km at high Reynolds numbers, the possibility arises of considerably simplifying the system of Navier–Stokes equations and of using the boundary-layer equations. Methods of calculating the three-dimensional laminar boundary layer[3] are used. One can obtain a solution of the system of equations of the three-dimensional boundary layer using the method of successive approximations in the case of equilibrium physical-chemical transitions and employ it to estimate the heat fluxes to the surface of bodies on the descending part of the trajectory.

## 1. The physical-mathematical model

We will write the system of equations of the three-dimensional boundary layer for a chemically equilibrium multicomponent gas ignoring radiation and thermal diffusion, introducing a curvilinear system of coordinates connected with the surface of the body around which the flow occurs: $\xi$ and $\eta$ are the coordinates on the surface, and the coordinate $\zeta$ is measured along the normal. The system of equations describing the flow of a multicomponent reactive mixture of perfect gases, includes the equations of continuity, momentum and energy of the mixture, the mass-balance equations of the components and the equation of state.

☆ *Prikl. Mat. Mekh.* Vol. 71, No. 5, pp. 827–836, 2007.
*E-mail address:* shevelev@icad.org.ru (Yu.D. Shevelev).

0021-8928/$ – see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jappmathmech.2007.11.007

The equation of continuity for a gas mixture as a whole has the form[3]

$$\frac{\partial}{\partial\xi}\left(\rho\sqrt{\frac{g}{g_{11}}}u\right)+\frac{\partial}{\partial\eta}\left(\rho\sqrt{\frac{g}{g_{22}}}\omega\right)+\sqrt{g}\frac{\partial\rho\upsilon}{\partial\zeta} = 0 \tag{1.1}$$

where ρ is the density of the mixture, u, ω and υ are the components of the velocity vector along the ξ, η and ζ axes respectively, and $g_{11}$ and $g_{22}$ are the components of the metric tensor in the basis of the curvilinear system of coordinates ξ, η, ζ.

The momentum equations in projections on the axes of coordinates in the boundary-layer approximation can be written in the form

$$\hat{L}u+A_1u^2+A_2\omega^2+A_3u\omega = \frac{A_4}{\rho}+\frac{1}{\rho}\frac{\partial}{\partial\zeta}\left(\mu\frac{\partial u}{\partial\zeta}\right) \tag{1.2}$$

$$\frac{\partial p}{\partial\zeta} = 0 \tag{1.3}$$

$$\hat{L}\omega+B_1u^2+B_2\omega^2+B_3u\omega = \frac{B_4}{\rho}+\frac{1}{\rho}\frac{\partial}{\partial\zeta}\left(\mu\frac{\partial\omega}{\partial\zeta}\right) \tag{1.4}$$

Here $\hat{L}$is the differential operator of the total (substantional) derivative along the ξ, η, ζ axes

$$\hat{L}=\frac{u}{\sqrt{g_{11}}}\frac{\partial}{\partial\xi}+\frac{\omega}{\sqrt{g_{22}}}\frac{\partial}{\partial\eta}+\upsilon\frac{\partial}{\partial\xi}$$

and μ is the coefficient of molecular viscosity of the mixture. The coefficients $A_k$ and $B_k$ are determined by the geometry of the body and the external flow; their form is known.[3]

The equation of conservation of energy can be represented as follows:

$$\begin{gathered}\hat{L}H = \frac{1}{\rho}\frac{\partial}{\partial\zeta}\left\{\frac{\mu}{\Pr}\left[\frac{\partial H}{\partial\zeta}+(\Pr-1)\frac{\partial}{\partial\zeta}\left(\frac{U^2}{2}\right)-\sum_i h_i\left(\frac{\Pr}{\mu}J_i+\frac{\partial c_i}{\partial\zeta}\right)\right]\right\}\\ H = h+\frac{U^2}{2},\quad h = \sum_i c_ih_i,\quad h_i = \int_0^T c_{pi}dT+h_i^0;\quad \Pr = \frac{c_p\mu}{\lambda},\quad c_p = \sum_i c_i\frac{dh_i}{dT} = \sum_i c_ic_{pi}\end{gathered} \tag{1.5}$$

Here $H$ is the total enthalpy, $c_i$ and $h_i^0$ are the mass concentration and specific heat of formation of the $i$-th component, $J_i$ is the projection of the diffusion flow of the $i$-th component onto the ζ axis, and Pr is the Prandtl number. The coefficient of viscosity μ and the thermal conductivity λ of the gas mixture are functions not only of the temperature but also of the mass concentrations of the components: $\mu=\mu(c_i,T)$ and $\lambda=\lambda(c_i,T)$.

The equations of continuity of the components of the gas mixture in the general case of chemically nonequilibrium flows can be written in the boundary-layer approximation as

$$\rho\hat{L}c_i+\frac{\partial J_i}{\partial\zeta}=\dot{w}_i,\quad i=1,\ldots,N$$

where $\dot{w}_i$ is the mass rate of formation of the $i$-th component due to chemical reactions.

The assumption of chemical equilibrium considerably simplifies the description of the flows. In this case the N-L equations of continuity are replaced by algebraic relations, which express the conditions of chemical equilibrium

$$\prod_{j=1}^{N}\left(\frac{c_jm}{m_j}\right)^{\nu'_j-\nu''_j} = \frac{K_{pi}(T)}{p^\alpha},\quad \alpha = \sum_{j=1}^{N}(\nu''_j-\nu'_j),\quad i = 1,2,\ldots,N-L \tag{1.6}$$

where $K_{pi}$ are the constants of equilibrium of the independent chemical reactions, $\nu'_j$, $\nu''_j$ are the stoichiometric coefficients of the reactants and the products of the chemical reactions, $p$ is the pressure, m is the molar mass of the mixture, calculated in terms of the molar masses of the components, and $L$ is the number of independent components (chemical elements). Using relations (1.6), the concentrations of the components of the mixture are determined in terms of its temperature, pressure and the concentration of the chemical elements. In the case of the equilibrium flow of a gas mixture of components with different diffusion coefficients, and also when there are thermal diffusion and barodiffusion effects, the concentrations of the elements are found, as before, from the corresponding equations of continuity.[4]

Further simplification of the mathematical description of chemically equilibrium flows is possible by using a binary model of the diffusion. In this approximation, which is used in the present paper, when there are no sources of the masses, the concentration of the chemical elements will be constant quantities, and diffusion flows of the elements will be equal to zero. In this case the chemical composition and the transfer coefficients of the equilibrium gas mixture are functions of two parameters, for example, the pressure and the temperature. In many cases this approximation gives completely acceptable results.

System of equations (1.1)–(1.6) is supplemented by the equation of state for a gas mixture

$$p = \rho\frac{R}{m}T, \quad m = \left(\sum_{i=1}^{N}\frac{c_i}{m_i}\right)^{-1} \tag{1.7}$$

The boundary conditions are

$$\zeta = 0: H = H_w,\ u_w = \omega_w = \upsilon_w = 0; \quad \zeta \to \infty: H = H_e,\ u = u_e,\ \omega = \omega_e,\ c_i = c_{ie}$$

In the approximation considered, the system of equations for the flows of a gas with equilibrium physical-chemical transitions differs from the analogous system for a non-reactive gas solely by the use of more complex relations for the internal energy, enthalpy and the transfer coefficients of the gas mixture, which will be not only functions of the temperature but of the pressure also.

## 2. Solution of the boundary-layer equations in the first approximation

To solve system of equations (1.1)–(1.7) we will introduce the new variable

$$\zeta_1 = \sqrt{\frac{u_e}{\mu_e\rho_e\xi}}\int_0^{\zeta}\rho d\zeta$$

We will change from the required functions $u$, $\omega$, $\upsilon$ and $H$ to new variables $E$, $G$, $K$, and $\theta$ by the formulae

$$u = u_e(\xi,\eta)E, \quad \omega = \eta u_e(\xi,\eta)[G + \varphi E]$$

$$\nu = \sqrt{\frac{u_e\rho_e u_e}{\xi}}\left[K - \frac{\xi}{\sqrt{g_{11}}}E\frac{\partial\zeta_1}{\partial\xi} - \frac{\xi\eta}{\sqrt{g_{22}}}(G + \varphi E)\frac{\partial\zeta_1}{\partial\eta}\right], \quad H = H_e + (H_e - H_w)\theta; \quad \varphi = \frac{\omega_e}{\eta u_e}$$

As a result of these changes, we obtain equations for the normalized functions $E$, $G$, $K$ and $\theta$. Integration of this system of equations twice with respect to $\zeta_1$ (first from $\zeta_1$ to $\infty$ and then from 0 to $\zeta$) gives a non-linear system of integro-differential equations, which can be solved by the method of successive approximations.

We will assume that we know the $n$-th approximation of the required functions $E^{(n)}$, $G^{(n)}$ and $\theta^{(n)}$. Substituting this approximation into the right-hand side of the integro-differential equations, we obtain the $(n+1)$-th approximation. In order that the $(n+1)$-th approximation should satisfy the boundary conditions, we introduce the control functions $\delta^{(n)}$, $b^{(n)}$ and $d^{(n)}$.

The system of equations for finding the $(n+1)$-th approximation can be written in the form

$$\begin{aligned} &E^{(n+1)} = \delta^{(n)}F_1^{(n)}, \quad G^{(n+1)} = \delta^{(n)}F_2^{(n)}, \quad \theta^{(n+1)} = \delta^{(n)}F_3^{(n)} \\ &F_k^{(n)} = A_{ak}^{(n)} + b^{(n)}B_{ak}^{(n)} + b^{(n)^2}C_{ak}^{(n)}, \quad k = 1, 2 \\ &F_3^{(n)} = A_{a3}^{(n)} + b^{(n)}B_{a3}^{(n)} + d^{(n)}C_{a3}^{(n)} + b^{(n)}d^{(n)}D_{a3}^{(n)} + T_{a3}^{(n)} \end{aligned} \tag{2.1}$$

The equations for the unknown control functions $\delta^{(n)}$, $b^{(n)}$ and $d^{(n)}$ are obtained from the system of equations (2.1) when $\zeta_1 \to \infty$:

$$\delta^{(n)}F_1^{(n)} = 1, \quad F_2^{(n)} = 0, \quad \delta^{(n)}F_3^{(n)} = 1 \tag{2.2}$$

The form of the coefficients $A_{ak}^{(n)}$, $B_{ak}^{(n)}$, ... is determined by the geometry of the body, the parameters of the external flow and the preceding approximation.[3] To calculate these coefficients it is also necessary to know the distribution of the parameters $\rho_e/\rho$, $1/l$, ($l = \mu\rho/(\mu_e\rho_e)$) across the boundary layer.

We will specify approximately the distribution of these parameters in terms of their values on the wall and on the outer boundary of the boundary layer in the form

$$\rho_e/\rho = 1+(\rho_e/\rho_w-1)z_{-1}^{\alpha_1}(\zeta_1), \quad 1/l = 1+(1/l_w-1)z_{-1}^{\alpha_2}(\zeta_1) \tag{2.3}$$

The functions $z_m(\zeta_1)$ belong to the class of functions in which the coefficients $A_m$ are chosen so that $z_m(0)=1$ and $z_{-1}(\zeta_1)=\exp(-\zeta_1^2)$:

$$z_m(\zeta_1) = \frac{A_m}{m!}\int_{\infty}^{\zeta_1}(\zeta_1-\xi)^m \exp(-\xi^2)d\xi$$

An analysis of two-dimensional equilibrium boundary layers[5] enables us to determine the approximate values of the parameters in expressions (2.3): $\alpha_1 = 1.2$ and $\alpha_2 = 1.4$.

The first approximation of the solution of the problem is obtained if we take the following expressions as the zeroth approximation

$$E^{(0)} = 1-z_0(\zeta_1),\ G^{(0)}=b^{(0)}[z_0(\zeta_1)-z_1(\zeta_1)],\ \theta^{(0)} = 1-z_0(\zeta_1)+d^{(0)} = 1-z_0(\zeta_1)+d^{(0)}[z_0(\zeta_1)-z_{-1}(\zeta_1)]$$

After $\delta^{(n)}$, $b^{(n)}$ and $d^{(0)}$ have been obtained from the algebraic system of equations (2.2), we can determine the functions $E^{(1)}$, $G^{(1)}$ and $\theta^{(1)}$ from Eqs. (2.1), and then also quantities proportional to the surface friction coefficients and the value of the heat flux

$$-l_w\frac{\partial E}{\partial\zeta_1}\bigg|_{\zeta_1=0} = \sqrt{\delta_0}\left\{N_1^*\left[0.808\left(1-\frac{\rho_e}{\rho_w}\right)-0.798\right]-0.234P_1^*+b^{(0)}(0.113P_2^*-0.209N_3^*)\right.$$
$$\left.+0.0709N_2^*b^{(0)^2}\right\} - l_w\frac{\partial G}{\partial\zeta_1}\bigg|_{\zeta_1=0} = \sqrt{\delta_0}\left\{M_1^*\left[0.808\left(1-\frac{\rho_e}{\rho_w}\right)-0.798\right]\right.$$
$$\left.-0.209b^{(0)}(P_1^*+M_3^*)+0.709b^{(0)^2}(P_2^*+M_2^*)\right\} - \frac{l_w}{Pr_w}\frac{\partial\theta}{\partial\zeta_1}\bigg|_{\zeta_1=0}$$
$$= \sqrt{\delta^{(0)}}[-P_1^*(0.234+0.209d^{(0)})+b^{(0)}P_2^*(0.113+0.0709d^{(0)}]$$

The coefficients $N_1^*$, $N_2^*$, $N_3^*$, $M_1^*$, $M_2^*$, $M_3^*$, $P_1^*$ and $P_2^*$ depend on the parameters of the external flow and on the geometry of the body. The form of these coefficients were given previously in the general case in Ref. 3.

When considering hypersonic flow around a body with spherical blunting, with a high degree of accuracy we can put $b^{(0)} \approx 0$.[3] If the components of the local friction coefficient is defined as follows:

$$c_{f1} = \frac{\mu_w}{\rho_e u_e^2}\left(\frac{\partial u}{\partial\zeta}\right)_{\zeta=0}, \quad c_{f2} = \frac{\mu_w}{\rho_e u_e^2}\left(\frac{\partial\omega}{\partial\zeta}\right)_{\zeta=0}$$

then, in this case,

$$\begin{aligned} c_{f1}\sqrt{\mathrm{Re}} &= l_w\frac{\partial E}{\partial\zeta_1}\bigg|_{\zeta_1=0} = \sqrt{\delta^{(0)}}\left\{0.234P_1^*+\left[0.798-0.808\left(1-\frac{\rho_e}{\rho_w}\right)\right]N_1^*\right\} \\ c_{f2}\sqrt{\mathrm{Re}} &= l_w\frac{\partial G}{\partial\zeta_1}\bigg|_{\zeta_1=0} = c_{f_1}\sqrt{\mathrm{Re}}\,\mathrm{tg}\gamma \end{aligned} \tag{2.4}$$

where $\mathrm{Re} = u_e\rho_e\xi/\mu_e$ is the local Reynolds number and $\gamma$ is the angle which the streamlines of the external flow make with the coordinate line $\eta = \mathrm{const}$, where $\mathrm{tg}\gamma = \omega_e/u_e$. Equilibrium physical-chemical transitions mainly affect the quantities $\rho_e$ and $\mu_e\rho_e$, which are calculated from the formulae for an equilibrium gas, and the quantities $\delta^{(0)}$ and $d^{(0)}$, which also depend on these properties of the gas.

The heat flux to the body surface for a multicomponent gas mixture is given by the expression

$$q_w = -\lambda\frac{\partial T}{\partial\zeta}+\sum_{i=1}^N h_iJ_i = -\frac{\mu}{\mathrm{Pr}}\frac{\partial H}{\partial\zeta}+\frac{\mu}{\mathrm{Pr}}\sum_i h_i\frac{\partial c_i}{\partial\zeta}+\sum_i h_iJ_i \tag{2.5}$$

(the last term in this chain of equalities is the result of changing from the temperature $T$ to the total enthalpy $H$). Introducing the Lewis number $\mathrm{Le}_i$, which represents the ratio of the molecular heat transfer to diffusion transfer, we have for the heat flux

$$q_w = -\frac{\mu}{\Pr}\left(\frac{\partial H}{\partial \zeta} + \sum(\mathrm{Le}_i - 1)h_i\frac{\partial c_i}{\partial \zeta}\right); \quad \mathrm{Le}_i = \frac{\rho D_i c_{pi}}{\lambda} \tag{2.6}$$

($D_i$ is the effective diffusion coefficient).

Changing to normalized functions we obtain

$$q_w = -\sqrt{\frac{\mu_e\rho_e u_e}{\xi}}\frac{l_w}{Pr}\left[\left(\frac{\partial\theta}{\partial\zeta_1}\right)_{\zeta_1=0} + R^*\right](H_e - H_w) \quad R^* = \sum_{i=1}^{N}\frac{h_{iw}(c_{ie}-c_{iw})}{H_e - H_w}(\mathrm{Le}_i - 1)\left.\frac{\partial\chi_i}{\partial\zeta_1}\right|_{\zeta_1=0},$$

$$\chi_i = \frac{c_i - c_{iw}}{c_{ie} - c_{iw}}$$

The quantity $R^*$ depends on the process of diffusion of the components in the boundary layer, which can be neglected if the Lewis number is equal to unity, or if the concentrations on the outer boundary of the layer and on the surface are equal.

Below we will confine ourselves to the case when the rates of heat transfer due to diffusion and thermal conduction are equal (when the Lewis number is equal to unity). Then, we obtain for the heat flux

$$q_w = -\sqrt{\frac{\mu_e\rho_e u_e}{\xi}}(H_e - H_w)\frac{l_w}{\Pr}\left.\frac{\partial\theta}{\partial\zeta_1}\right|_{\zeta_1=0} \tag{2.7}$$

The dimensionless heat flux in the case of hypersonic flow around the body can be written in the form

$$\frac{l_w}{\Pr}\left.\frac{\partial\theta}{\partial\zeta_1}\right|_{\zeta_1=0} = \sqrt{\delta^{(0)}}P_1^*(0.234 + 0.209d^{(0)}) \tag{2.8}$$

The quantities $\delta^{(0)}$ and $d^{(0)}$ are calculated from the formulae

$$\delta^{(0)} = [0.399P_1^* - (0.149P_1^* - 0.006N_1^*)/l_w - N_1^*(0.318/l_w + 0.099)\rho_e/\rho_w]^{-1}$$

$$d^{(0)} = \frac{1-(1-\Pr)/[k(1-t_w)]}{\delta^{(0)}P_1^*\Pr(0.152+0.158/l_w)} - \frac{0.0973+0.153/l_w}{0.152+0.158/l_w}, \quad t_w = \frac{h_w}{H_e}, \quad k = \frac{2H_e}{U_e^2} \tag{2.9}$$

$$N_1^* = \frac{\xi}{u_e}\frac{\partial u_e}{\partial\xi} + \frac{\omega_e\xi}{u_e^2}\frac{\partial u_e}{\partial\eta}, \quad 2P_1^* = 1 + \frac{\xi}{g_{22}}\frac{\partial g_{22}}{\partial\xi} + \frac{\xi}{u_e}\frac{\partial u_e}{\partial\xi} + \frac{\xi}{\rho_e\mu_e}\frac{\partial(\rho_e\mu_e)}{\partial\xi}$$

Hence, knowing the distribution of the parameters of the gas mixture on the outer boundary of the boundary layer, we can determine the friction and heat flux on the surface of the body from the formulae obtained.

The algorithm for calculating the hypersonic flow of a chemically equilibrium gas flow round a blunt body is as follows. When calculating the external flow we use data for the pressure and velocity in a uniform perfect gas, obtained from the solution of Euler's equations. The density, temperature and composition of the mixture on the outer boundary of the boundary layer are found from the solution of the following system: the equation of conservation of total enthalpy $H_e - \sum_i h_{ie}c_{ie} - U_e^2/2 = 0$, the equations of chemical equilibrium (1.6), the equation of state of the gas mixture (1.7) and the equations of conservation of particles. Hence, we determined the composition of the mixture $c_{ie}$, the density, the temperature and the outer boundary of the boundary layer, and from formulae (2.4) and (2.7)–(2.9) we obtain the friction and heat exchange parameters on the body surface

It should be noted that the above formulae have singularities at the stagnation point of the flow when $\xi \to 0$. In this connection it is necessary to consider separately the problem of determining the parameters of the friction and of the convective heat flux in the neighbourhood of the critical point.

## 3. Estimate of the heat flux in the neighbourhood of the critical point

Consider the flow in the neighbourhood of the critical point of a sphere. The system of coordinates is chosen with origin at the stagnation point of the flow ($\xi = \eta = \zeta = 0$). We know that the velocity distribution in the outer flow in the region of the critical point will be linear ($u_e \sim \xi$, $\omega_e \sim \eta$). Taking this into account we can analyse the parameters which define the heat flux at a short distance from the stagnation point, determine the limiting values of the derivatives of the velocities with respect to the coordinates, and we can then extrapolate the results into the region of the stagnation point.

By formula (2.8), the heat flux at the critical point can be written in the form

$$q_k = -(\rho_e\mu_e)^{1/2}\left(\frac{\partial u_e}{\partial \xi}\right)_k^{1/2}(H_e - H_w)\frac{l_w}{\Pr}\frac{\partial\theta}{\partial\zeta_1}\bigg|_{\zeta_1=0} \tag{3.1}$$

Taking into account the singularities of the velocity distribution in the region of the critical point, when $\xi, \eta \to 0$ we can estimate the quantities $N_1^*$, $P_1^*$, $\delta^{(0)}$, $d^{(0)}$ which define the heat-transfer parameter $(l_w/Pr)(\partial\theta/\partial\zeta_1)_{\zeta_1=0}$. It was established by calculations that in the case of hypersonic flow with a Prandtl number Pr = 0.6–0.7 the value of this parameter at the critical point of a sphere is equal to unity to within 10%.

Hence, the formula for the heat flux at the stagnation point of the flow has the form

$$q_k \approx -(\rho_e\mu_e)^{1/2}(\partial u_e/\partial\xi)_k^{1/2}(H_e - H_w) \tag{3.2}$$

We will compare the formula obtained with the well-known correlation formula[5] for the heat flux at the critical point in the case of an equilibrium boundary layer. In the special case considered here (a Lewis number equal to unity and a Prandtl number Pr = 0.65) this formula will have the form

$$q_k \approx -0.99(\rho_e\mu_e)^{0.4}(\partial u_e/\partial\xi)_k^{1/2}(\rho_w\mu_w)^{0.1}(H_e - H_w) \tag{3.3}$$

We will assume, with a sufficient degree of accuracy, that $(\rho_w\mu_w)^{0.1}/(\rho_e\mu_e)^{0.1} \approx 1$, and hence we note the good agreement between formulae (3.2) and (3.3).

We will convert formula (3.2) for the heat flux to a form which is more convenient for calculations.

The quantity $(\partial u_e/\partial\xi)_k$ can be estimated from a consideration of the friction in the neighbourhood of the critical point[6] and can be written as follows:

$$(\partial u_e/\partial\xi)_k = R^{-1}\sqrt{2(p_e - p_\infty)/\rho_e} \tag{3.4}$$

where $R$ is the radius of blunting of the body and $p_\infty$ is the free-stream pressure.

It was shown in Ref. 6 that for approximate modelling of flow with a complex equation of state one can use the model of a perfect gas with constant adiabatic index $\gamma_*$. In the case of equilibrium flow the effective adiabatic index $\gamma_*$ is found from the condition $\rho_e/\rho_\infty = (\gamma_* + 1)(\gamma_* - 1)$ and determines the compression at a direct shock wave. Note also that at the critical point $p_e \approx \rho_\infty V_\infty^2 \gg p_\infty$. With these assumptions relation (3.4) can be written in the form

$$\left(\frac{\partial u_e}{\partial\xi}\right)_k = \frac{V_\infty}{R}\left(2\frac{\gamma_* - 1}{\gamma_* + 1}\right)^{1/2} \tag{3.5}$$

For practical purposes one can usually use approximate formulae for the temperature dependence of the coefficient of viscosity (for example, a power relation). As calculations show, when considering equilibrium dissociating gaseous mixtures one can use the following approximation to estimate the value of the viscosity (for air $n = 0.3$):

$$\mu \approx C_0 h^{-n} p/\rho \tag{3.6}$$

In this connection, assuming that, at the critical point $h_e \approx V_\infty^2/2$, we obtain

$$\mu_e\rho_e = \frac{P_e h_\infty^n}{P_\infty h_e^n}\mu_\infty\rho_\infty = \left(\frac{2\gamma}{\gamma - 1}\right)^n\left(\frac{V_\infty^2}{R_* T_\infty}\right)^{1-n}\mu_\infty\rho_\infty \tag{3.7}$$

Here $R_*$ is the specific gas constant and $\gamma$ is the free-stream adiabatic index.

At hypersonic velocities

$$H_e - H_w \approx V_\infty^2(1 - t_w)/2 \tag{3.8}$$

As a result of all the transformations, relation (3.2) takes the form

$$q_k = \left(\frac{\rho_\infty}{R}\right)^{1/2} V_\infty^{7/2-n}\frac{\mu_\infty^{1/2}}{(R_* T_\infty)^{(1-n)/2}}C_1 C_2(1 - t_w),\quad C_1 = \left(\frac{\gamma_* - 1}{\gamma_* + 1}\right)^{1/4},\quad C_2 = 2^{-3/4}\left(\frac{2\gamma}{\gamma - 1}\right)^{n/2}(3.9) \tag{3.9}$$

Table 1

| $H$, km | $V$, km/s | Formula | $q_k$, W/cm$^2$ | | |
|---|---|---|---|---|---|
| | | (3.11) | [8] | [9] | [10] |
| 20 | 3 | 130 | 140 | 140 | 150 |
| 30 | 4 | 150 | 150 | 160 | 160 |
| 40 | 5 | 150 | 140 | 150 | 150 |
| 50 | 6 | 130 | 120 | 140 | 140 |
| 60 | 6 | 75 | 70 | 80 | 80 |

The quantity $C_1$ varies from 0.47 to 0.64 for a change in the effective adiabatic index $\gamma_*$ from 1.1 to 1.4. Taking the properties of a real gas into account,[7] we can assume with a sufficient degree of accuracy that $\gamma_* = 1.2$.

Taking $n = 0.3$ in formula (3.6), we have for the heat flux at the critical point

$$q_k = 0.44\left(\frac{\rho_\infty}{R}\right)^{1/2} V_\infty^{3.2} \frac{\mu_\infty^{1/2}}{(R_* T_\infty)^{0.35}}(1 - t_w), \text{ Вт/м}^2 \tag{3.10}$$

All the dimensional quantities in formula (3.10) are expressed in the SI system ($R$ is in metres, $\rho_\infty$ is in kg/m$^3$, $V_\infty$ is in metres per second, $\mu_\infty$ in N s/m$^2$, $T_\infty$ is in degrees K, and $R_*$ is in J/(kg degree)).

In the case of the Earth's atmosphere, expressing R in metres, $\rho_\infty$ in kg/m$^3$ and $V_\infty$ in metres per second, we obtain

$$q_k \approx 3.3 \times 10^{-5}\left(\frac{\rho_\infty}{R}\right)^{1/2} V_\infty^{3.2}(1 - t_w), \quad \text{Вт/м}^2 \tag{3.11}$$

For the atmosphere of Mars the factor 3.3 is changed to 3.5 in the analogous formula.

The results of a calculation using formula (3.11) for the Earth's atmosphere and the analogous formula for the atmosphere of Mars were compared with the calculations of other researchers.[8–10]

In Table 1 we show the values of the heat flux at the critical point of a sphere of radius $R = 1$ m for different values of the altitudes $H$ and flight velocities $V$ in the Earth's atmosphere in the case of a "cold" surface ($t_w = 0$). The disagreement between the corresponding data in the table does not exceed 15%.

For the MARS EXPRESS probe (the frontal surface of the probe is a spherically blunted cone with an aperture angle of $2\pi/3$ and a blunting radius $R = 0.38$ m) the values of the heat flux $q_k$ at the critical point as a function of the Reynolds number, constructed from the free-stream parameters in the atmosphere of Mars, are presented in Fig. 1, according to the results obtained (curve 1) and the data calculated from the Sutton and Graves formula[8] (curve 2).

In Fig. 2 we show curves of the relative heat flux $q/q_k$ against the coordinate s, measured from the critical point along the surface of the MARS EXPRESS probe at a flight altitude $H = 43$ km in the atmosphere of Mars. The results

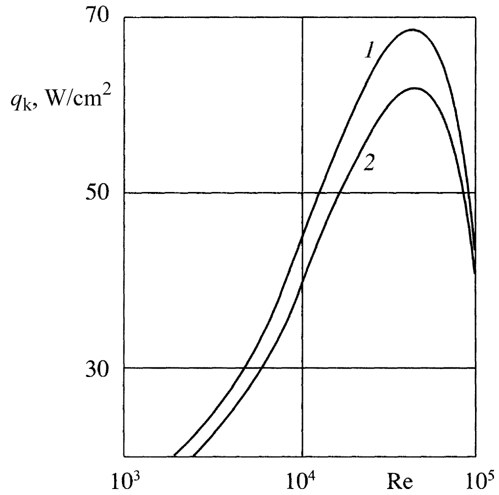


Fig. 1.

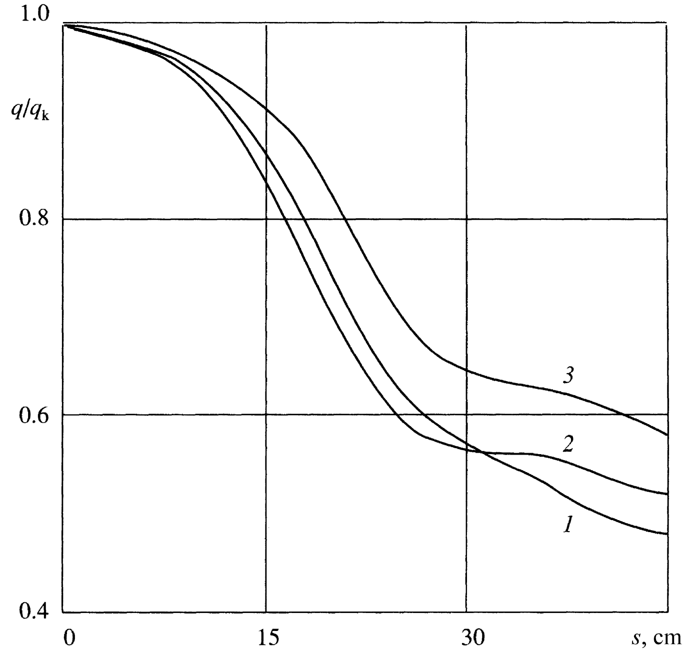


Fig. 2.

of the calculation using the method of successive approximations (curve 1) are compared with the corresponding data obtained using numerical methods to solve the Navier–Stokes equations[11,12] (curves 2 and 3 respectively). The distributions of $q/q_k$, obtained by the different methods, agree satisfactorily with one another.

This research was supported financially by the International Association for promoting Cooperation with Scientist from the New Independent States of the Former Soviet Union (INTAS-03-51-5204).

## References

1. Tirskiy GA. Continual models in problems of the hypersonic flow of a rarefied gas around blunt bodies. *Prikl Mat Mekh* 1997;**61**(6):903–30.
2. Brykina IG, Rogov BV, Tirskiy GA. Continual models of rarefied gas flows in problems of hypersonic aerodynamics. *Prikl Mat Mekh* 2006;**70**(6):995–1025.
3. Shevelev YuD. *Three-Dimensional Problems of the Theory of the Laminar Boundary Layer*. Moscow: Nauka; 1977.
4. Pilyugin NN, Tirskiy GA. *The Dynamics of an Ionized Radiating Gas*. Moscow: Izd MGU; 1989.
5. Fay JA, Riddell FR. Theory of stagnation point heat transfer in dissociated air. *J Aeronaut Sci* 1958;**25**(2):73–85.
6. Stulov VP, Mirskii VN, Vislyi AN. *The Aerodynamics of Fire-Balls*. Moscow: Nauka. Fizmatlit; 1995.
7. Lebedev MT, Minonostsev VB, Telenin GF, Tinyakov TP. An approximate method of taking into account the real nature of a gas in hypersonic flow around segmental bodies. *Izv Akad Nauk SSSR MZhG* 1969;**2**:107–11.
8. Sutton J, Graves RA. A general stagnation point convective heating equation for arbitrary gas mixtures. *NASA TR* 1972;**R-376**.
9. Martin JJ. *Atmospheric Reentry An Introduction to its Science and Engineering*. Old Tappan, NJ: Prentic-Hall; 1966.
10. Avduyevskii VS, Galitseiskii BM, Glebov GA, et al. *Principles of Heat Transfer in Aviation and Space-Rocket Techniques*. Moscow: Mashinostroyeniye; 1975.
11. Shevelev Yu D, Maksimov FA, Syzranova NG. Numerical modelling of the supersonic flow of reacting gas mixtures around bodies. In: *Proceedings of the 14th International Conference on Computational Mechanics in Modern Applied Program Systems*. Alushta: Krym; 2005.
12. Afonina NYe, Gromov VG. Numerical modelling of the flow around the Mars spacecraft. *Aeromekh Gaz Dynamika* 2001;**2**:35–47.

Translated by RCG